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QCD Sum Rule Calculation
of $\gamma\gamma^* \to \pi^0$ Transition Form Factor

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Abstract

We develop a QCD sum rule analysis of the form factor $F_{\gamma^*\gamma^*\pi^\circ}(q^2, Q^2)$ in the region where virtuality of one of the spacelike photons is small $q^2 \ll 1\,\mathrm{GeV}^2$ while another is large: $Q^2 \gtrsim 1\,\mathrm{GeV}^2$. We construct the operator product expansion suitable for this kinematic situation and obtain a QCD sum rule for $F_{\gamma^*\gamma^*\pi^\circ}(0, Q^2)$. Our results confirm expectation that the momentum transfer dependence of $F_{\gamma^*\gamma^*\pi^\circ}(0, Q^2)$ is close to interpolation between its $Q^2 = 0$ value fixed by the axial anomaly and Q^{-2} pQCD behaviour for large Q^2. Our approach, in contrast to pQCD, does not require additional assumptions about the shape of the pion distribution amplitude $\varphi_\pi(x)$. The absolute value of the $1/Q^2$ term obtained in this paper favours $\varphi_\pi(x)$ close to the asymptotic form $\varphi_\pi^{as}(x) = 6f_\pi x(1 - x)$.

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1. Introduction.

The transition $\gamma^*\gamma^* \to \pi^0$ of two virtual photons γ^* into a neutral pion provides an exceptional opportunity to test QCD predictions for exclusive processes. In the lowest order of perturbative QCD, its asymptotic behaviour is due to the subprocess $\gamma^*(q_1) + \gamma^*(q_2) \to \bar{q}(\bar{x}p) + q(xp)$ with x $(\bar{x})$ being the fraction of the pion momentum p carried by the quark produced at the q_1 (q_2) photon vertex (see Fig.1a). The relevant diagram is similar to the handbag diagram for deep inelastic scattering, with the main difference that one should use the pion distribution amplitude $\varphi_\pi(x)$ instead of parton densities. For large Q^2, the perturbative QCD prediction is given by [1]:

$$F^{as}_{\gamma^*\gamma^*\pi^0}(q^2, Q^2) = \frac{4\pi}{3} \int_0^1 \frac{\varphi_\pi(x)}{xQ^2 + \bar{x}q^2} \, dx \xrightarrow{q^2=0} \frac{4\pi}{3} \int_0^1 \frac{\varphi_\pi(x)}{xQ^2} \, dx \equiv \frac{4\pi}{3Q^2} I \tag{1}$$

$(Q^2 \equiv -q_2^2,\ q^2 \equiv -q_1^2$ and our convention is $q^2 \leq Q^2)$. Experimentally, the most important situation is when one of the photons is almost real $q^2 \approx 0$. In this case, necessary nonperturbative information is accumulated in the same integral I (see eq.(1)) that appears in the one-gluon-exchange diagram for the pion electromagnetic form factor [2, 3, 4]. The value of I depends on the shape of the pion distribution amplitude $\varphi_\pi(x)$. In particular, using the asymptotic form $\varphi_\pi^{as}(x) = 6f_\pi x\bar{x}$ [2, 3] gives $F^{as}_{\gamma\gamma^*\pi^0}(Q^2) = 4\pi f_\pi/Q^2$ for the asymptotic behaviour [1]. If one takes the Chernyak-Zhitnitsky form $\varphi_\pi^{CZ}(x) = 30f_\pi x(1-x)(1-2x)^2$ [5], the integral I increases by a sizable factor of 5/3, and this difference can be used for experimental discrimination between the two forms.

   An important point is that, unlike the case of the pion EM form factor, the pQCD hard scattering term for $\gamma\gamma^* \to \pi^0$ (γ denoting a real photon) has zeroth order in the QCD coupling constant α_s, *i.e.*, the asymptotically leading term has no suppression. The situation is similar to that in deep inelastic scattering. Hence, we have good reasons to expect that pQCD for $F_{\gamma\gamma^*\pi^0}(Q^2)$ may work at accessible Q^2. Of course, the asymptotic $1/Q^2$-behaviour cannot be true in the low-Q^2 region, since the $Q^2 = 0$ limit of $F_{\gamma\gamma^*\pi^0}(Q^2)$ is known to be finite and normalized by the $\pi^0 \to \gamma\gamma$ decay rate. Using PCAC and ABJ anomaly [6], one can calculate $F_{\gamma\gamma^*\pi^0}(0)$ theoretically: $F_{\gamma\gamma^*\pi^0}(0) = 1/\pi f_\pi$. It is natural to expect that a complete QCD result does not strongly deviate from a simple interpolation $\pi f_\pi F_{\gamma\gamma^*\pi^0}(Q^2) = 1/(1 + Q^2/4\pi^2 f_\pi^2)$ [7] between the $Q^2 = 0$ value and the large-Q^2 asymptotics[3]. This interpolation implies the asymptotic form of the distribution amplitude for the large-Q^2 limit and agrees with CELLO experimental data [9]. It was also claimed [10] that the new CLEO data available up to $8\,GeV^2$ also agree with the interpolation formula. This provides a strong evidence that the pion distribution amplitude is rather close to its asymptotic form. Because of the far-reaching consequences of this conclusion, it is desirable to have a direct QCD calculation of the $\gamma\gamma^* \to \pi^0$ form factor in the intermediate region of moderately large momentum transfers $Q^2 \gtrsim 1\,GeV^2$. Such an approach is provided by QCD sum rules. As we will see below, the QCD sum rules also allow one to calculate $F_{\gamma\gamma^*\pi^0}(Q^2)$ for large Q^2 without any assumptions about the shape of the pion distribution amplitude. In fact,

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[3]In particular, such an interpolation agrees with the results of a constituent quark model calculation [8]

the QCD sum rule for $F_{\gamma\gamma^*\pi^0}(Q^2)$ can be used to get information about $\varphi_\pi^{as}(x)$.

2. Definitions.

The $\gamma^*\gamma^* \to \pi^0$ transition form factor $F_{\gamma^*\gamma^*\pi^0}(q^2, Q^2)$ can be defined through the matrix element

$$\int \langle \pi, \overrightarrow{p} \, | T \{ J_\mu(X) \, J_\nu(0) \} \, | 0 \rangle e^{-iq_1 X} d^4 X = \alpha\sqrt{2}\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \, F_{\gamma^*\gamma^*\pi^\circ}\left(q^2, Q^2\right), \qquad (2)$$

where $\alpha = e^2/4\pi$ is the fine structure constant, $J_\mu = e\left(\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d\right)$ is the electromagnetic current of the light quarks and $|\pi, \overrightarrow{p}\rangle$ is a π^0 state with the 4-momentum p. To incorporate QCD sum rules [11], we consider a three-point correlation function

$$\mathcal{F}_{\alpha\mu\nu}(q_1, q_2) = \frac{i}{\alpha\sqrt{2}} \int \langle 0|T \left\{ j_\alpha^5(Y) J_\mu(X) \, J_\nu(0) \right\} |0\rangle e^{-iq_1 X} \, e^{ipY} d^4 X \, d^4 Y \,, \qquad (3)$$

(cf. [12]) containing the axial current $j_\alpha^5 = \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_5\gamma_\alpha u - \bar{d}\gamma_5\gamma_\alpha d\right)$ serving as a field with a non-zero projection onto the neutral pion state: $\langle 0|j_\alpha^5(0)|\pi^0, \overrightarrow{p}\rangle = -i\, f_\pi p_\alpha$. The three-point amplitude $\mathcal{F}_{\alpha\mu\nu}(q_1, q_2)$ has a pole for $p^2 = m_\pi^2$:

$$\mathcal{F}_{\alpha\mu\nu}(q_1, q_2) = \frac{f_\pi}{p^2 - m_\pi^2} p_\alpha \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta F_{\gamma^*\gamma^*\pi^\circ}(q^2, Q^2) + \dots \,, \qquad (4)$$

i.e., the Lorentz structure of the pion contribution is $p_\alpha \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta$, and the spectral density of the dispersion relation

$$\mathcal{F}\left(p^2, q^2, Q^2\right) = \frac{1}{\pi} \int_0^\infty \frac{\rho\left(s, q^2, Q^2\right)}{s - p^2} \, ds + \text{``subtractions''}. \qquad (5)$$

for the relevant invariant amplitude can be written as

$$\rho\left(s, q^2, Q^2\right) = \pi f_\pi \delta(s - m_\pi^2) F_{\gamma^*\gamma^*\pi^\circ}\left(q^2, Q^2\right) + \text{``higher states''}. \qquad (6)$$

The higher states include A_1 and higher broad pseudovector resonances. Due to asymptotic freedom, their sum, for large s, rapidly approaches the pQCD spectral density $\rho^{PT}(s, q^2, Q^2)$. The simplest model is to approximate all the higher states, including the A_1, by the perturbative contribution:

$$\rho^{mod}\left(s, q^2, Q^2\right) = \pi f_\pi \delta(s) F_{\gamma^*\gamma^*\pi^\circ}\left(q^2, Q^2\right) + \theta(s - s_o)\rho^{PT}(s, q^2, Q^2) \qquad (7)$$

where the parameter s_o is the effective threshold for higher states. To suppress the higher states by an exponential weight $\exp[-s/M^2]$, we apply the SVZ-Borel transformation [11]:

$$\hat{B}(-p^2 \to M^2)\mathcal{F}(p^2, q^2, Q^2) \equiv \Phi(M^2, q^2, Q^2) = \frac{1}{\pi M^2} \int_0^\infty e^{-s/M^2} \rho(s, q^2, Q^2) \, ds, \qquad (8)$$
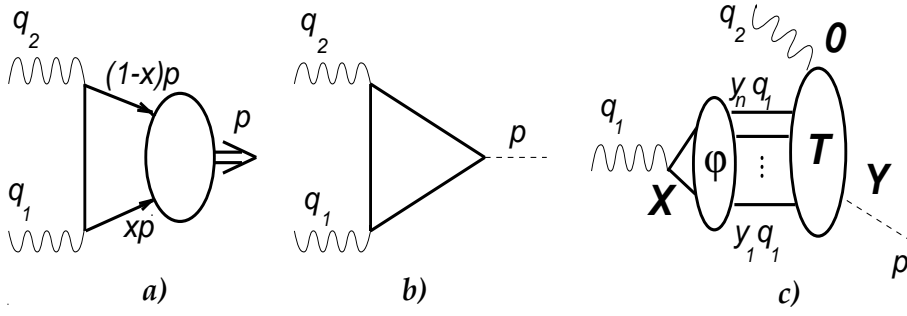
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Figure 1: *a)* Leading-order pQCD contribution. *b)* Triangle diagram. *c)* General structure of bilocal contributions.

which, moreover, produces a factorially improved OPE power series for large M^2: $1/(-p^2)^N \to (1/M^2)^N/(N-1)!$.

To construct a QCD sum rule, one should calculate the three-point function $\mathcal{F}(p^2, q^2, Q^2)$ and then its SVZ-transform $\Phi(M^2, q^2, Q^2)$ as a power expansion in $1/M^2$ for large M^2. However, a particular form of the original $(1/p^2)^N$-expansion depends on the values of the photon virtualities q^2 and Q^2.

3. QCD sum rules for large q^2.

The simplest case is when both virtualities are large: $Q^2 \sim q^2 \sim -p^2 > \mu^2$ where μ is some scale $\mu^2 \sim 1\,GeV^2$ above which one can rely on asymptotic freedom of QCD. Then all contributions which have a power behaviour $(1/M^2)^N$ correspond to a situation with all three currents close to each other: all the intervals $X^2, Y^2, (X-Y)^2$ are small.

The starting point of the OPE is the perturbative triangle graph (Fig.1*b*). Using Feynman parameterization and performing simple integrations we get:

$$\rho^{PT}(s, q^2, Q^2) = 2 \int_0^1 \frac{x\bar{x}(xQ^2 + \bar{x}q^2)^2}{[sx\bar{x} + xQ^2 + \bar{x}q^2]^3}\, dx \,. \tag{9}$$

The variable x is the light-cone fraction of the total pion momentum p carried by one of the quarks. Adding the condensate corrections, we obtain the following QCD sum rule:

$$\pi f_\pi F_{\gamma^*\gamma^*\pi^\circ}(q^2, Q^2) = 2 \int_0^{s_o} ds\, e^{-s/M^2} \int_0^1 \frac{x\bar{x}(xQ^2 + \bar{x}q^2)^2}{[sx\bar{x} + xQ^2 + \bar{x}q^2]^3}\, dx$$

$$+ \frac{\pi^2}{9}\langle\frac{\alpha_s}{\pi}GG\rangle \left(\frac{1}{2M^2Q^2} + \frac{1}{2M^2q^2} - \frac{1}{Q^2q^2}\right)$$

$$+ \frac{64}{243}\pi^3\alpha_s\langle\bar{q}q\rangle^2 \left(\frac{1}{M^4}\left[\frac{Q^2}{q^4} + \frac{9}{2q^2} + \frac{9}{2Q^2} + \frac{q^2}{Q^4}\right] + \frac{9}{Q^2q^4} + \frac{9}{Q^4q^2}\right). \tag{10}$$

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It is valid when both virtualities of the photons are large. In this region, the perturbative QCD approach is also expected to work. This expectation is completely supported by our sum rule. Indeed, neglecting the $sx\bar{x}$-term compared to $xQ^2 + \bar{x}q^2$ and keeping only the leading $O(1/Q^2)$ and $O(1/q^2)$ terms in the condensates, we can write eq.(10) as

$$F_{\gamma^*\gamma^*\pi^\circ}(q^2, Q^2) = \frac{4\pi}{3f_\pi} \int_0^1 \frac{dx}{(xQ^2 + \bar{x}q^2)} \left\{ \frac{3M^2}{2\pi^2}(1 - e^{-s_0/M^2})x\bar{x} \right.$$
$$+ \frac{1}{24M^2}\langle\frac{\alpha_s}{\pi}GG\rangle[\delta(x) + \delta(\bar{x})]$$
$$\left. + \frac{8}{81M^4}\pi\alpha_s\langle\bar{q}q\rangle^2\left(11[\delta(x) + \delta(\bar{x})] + 2[\delta'(x) + \delta'(\bar{x})]\right)\right\}. \tag{11}$$

The expression in curly brackets coincides with the QCD sum rule for the pion distribution amplitude $f_\pi\varphi_\pi(x)$ (see, e.g., [13]). Hence, when q^2, the smaller of two photon virtualities is large, the QCD sum rule (10) exactly reproduces the pQCD result (1).

4.   Operator product expansion for small q^2.

One may be tempted to get a QCD sum rule for the integral I by taking $q^2 = 0$ in eq.(10). Such an attempt, however, fails immediately because of the power singularities $1/q^2$, $1/q^4$, etc. in the condensate terms. It is easy to see that these singularities are produced by the $\delta(x)$ and $\delta'(x)$ terms in eq.(11). In fact, it is precisely these terms that generate the two-hump form for $\varphi_\pi(x)$ in the CZ-approach [5]. Higher condensates would produce even more singular $\delta^{(n)}(x)$ terms. As shown in ref.[13], the $\delta^{(n)}(x)$ terms result from the Taylor expansion of nonlocal condensates like $\langle\bar{q}(0)q(Z)\rangle$. Modelling nonlocal condensates by functions decreasing at large $(-Z^2)$, i.e., assuming a finite correlation length $\sim 1/\mu$ for vacuum fluctuations, one obtains smooth curves instead of the singular $\delta(x)$ and $\delta'(x)$ contributions, and the result for $\varphi_\pi(x)$ is close to the asymptotic form [13]. Effectively, the correlation length provides an IR cut-off in the end-point regions $x \sim 0, x \sim 1$. Similarly, we expect that, when q^2 is too small to provide an appropriate IR cut-off in the sum rule (10), such a cut-off is again generated by nonperturbative effects, i.e., that eventually $1/q^2$ is substituted for small q^2 by something like $1/m_\rho^2$. Below, we show that this is exactly what happens in the QCD sum rule framework.

To illustrate the nature of the modifications required in the small-q^2 region, it is instructive to analyze first the perturbative term. The latter, though finite for $q^2 = 0$, contains contributions which are non-analytic at this point:

$$\Phi^{PT}(q^2, Q^2, M^2) = \frac{1}{\pi M^2} \int_0^\infty e^{-s/M^2} \left\{ 1 + \left[2\frac{q^2y}{M^2} \right. \right.$$
$$\left. \left. + \frac{q^4y^2}{M^4}\right] e^{q^2y/M^2} \ln\left(\frac{q^2y}{M^2}\right) + \ldots \right\} \frac{Q^2ds}{(s + Q^2)^2}, \tag{12}$$

where $y = s/(s + Q^2)$ and dots stand for terms analytic and vanishing for $q^2 = 0$. The logarithms here are a typical example of mass singularities (see, e.g., [14, 15] and, for QCD sum rule applica-
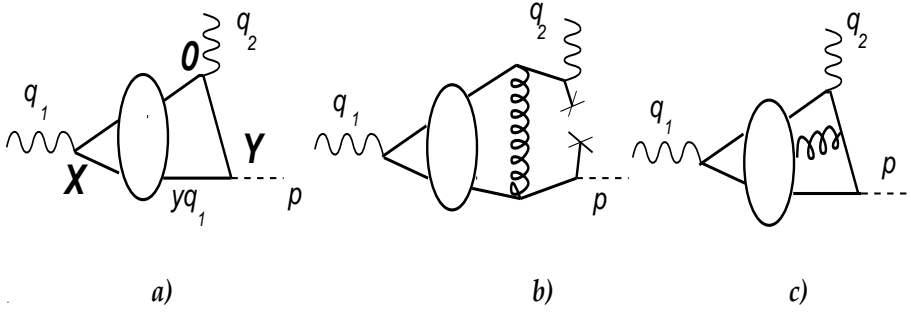
Figure 2: Bilocal contributions with coefficient functions given by *a)* single propagator; *b)* product of 3 propagators; *c)* product of 2 propagators.

tions, refs. [16, 17, 18]): the singularities are due to the possibility of the long-distance propagation in the q-channel. In other words, when q^2 is small, there appears an additional possibility to get a power-behaved contribution from a configuration in which the large momentum flows from the Q-vertex into the p-vertex (or *vice versa*) without entering the q-vertex, and small momenta flowing through other parts of the diagrams induce singular contributions. In the coordinate representation, such a configuration can be realized by keeping the electromagnetic current $J_\mu(X)$ of the low-virtuality photon far away from two other currents, which are still close to each other. The contribution generated in this regime can be extracted through an operator product expansion for the short-distance-separated currents: $T\{J(0)j^5(Y)\} \sim \sum C_i(Y)\mathcal{O}_i$. Diagrammatically, the situation is similar to the pQCD limit $q^2, Q^2 \gg -p^2$ discussed above. The only difference is that we should consider now the limit $-p^2, Q^2 \gg q^2$. The result again can be written in a "parton" form (see Fig.1c):

$$\mathcal{F}^{bilocal}(q_1, q_2, p) = \int_0^1 \phi_\gamma^{(i)}(\{y\}, q^2) T^{(i)}(\{yq_1\}; q_2, p)[dy], \tag{13}$$

where $\phi_\gamma^{(i)}(\{y\}, q^2)$ can be treated as distribution amplitudes of the q_1-photon, with y's being the light-cone fractions of the momentum q_1 carried by the relevant partons (*i.e.*, quark and gluonic G-fields present in $\mathcal{O}$). The functions $\phi_\gamma^{(i)}(\{y\}, q^2)$ are related to the correlators ("bilocals", cf. [19])

$$\Pi^{(i)}(q_1) \sim \int e^{iq_1 X}\langle 0|T\{J_\mu(X)\mathcal{O}^{(i)}(0)\}|0\rangle d^4X \tag{14}$$

of the $J_\mu(X)$-current with composite operators $\mathcal{O}^{(i)}(0)$. Performing such a factorization for each diagram, we represent the amplitude $\mathcal{F}$ as a sum of its purely short-distance (SD) and bilocal (B) parts. The SD-part (which is defined as the difference between the original unfactorized expression and the perturbative version of its B-part) is regular in the $q^2 \to 0$ limit and can be treated perturbatively. On the other hand, the low-q^2 behaviour of the B-correlators $\Pi(q_1)$ cannot be directly calculated in perturbation theory.

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5.   Structure of bilocal contributions.

In the simplest case, the amplitude $T^{(i)}(\{yq_1\}; q_2, p)$ in the bilocal term is given by a single quark propagator (Fig.2a):

$$T(yq_1, \bar{y}q_1; q_2, p) \sim \frac{1}{(p - yq_1)^2} = \frac{1}{\bar{y}p^2 + y\bar{y}q^2 - yQ^2}, \qquad (15)$$

accompanied by two-body distribution amplitudes $\phi_\gamma^{(i)}(y, q^2)$, with yq_1 and $\bar{y}q_1$ being the momenta carried by the quarks ($\bar{y} \equiv 1 - y$). The y^n-moments of $\phi_\gamma^{(i)}(y, q^2)$ are given by bilocals $\Pi_n^{(i)}(q^2)$ involving composite operators $\mathcal{O}_n^{(i)}$ with n covariant derivatives. Note, that it is legitimate to keep the q^2-term in eq.(15) when substituting it into eq.(13): all the $(q^2/Q^2)^N$ and $(q^2/p^2)^N$ power corrections are exactly reproduced there due to a phenomenon analogous to the ξ-scaling [20] in deep inelastic scattering. Applying the SVZ-Borel transformation, we obtain the result

$$\Phi_1^B(q^2, Q^2, M^2) \sim \frac{1}{M^2} \int_0^1 \phi_\gamma(y, q^2) e^{yq^2/M^2} e^{-yQ^2/\bar{y}M^2} dy, \qquad (16)$$

which has the structure of eq.(12): one should just take $yQ^2/\bar{y} = s$.

In perturbation theory, the amplitudes $\phi_\gamma^{(i)}(y, q^2)$ have logarithmic non-analytic behaviour for $q^2 = 0$. Eq.(12) indicates that, for the triangle graph, there are only two independent sources of logarithmic singularities. They correspond to two-body operators of leading and next-to-leading twist in the OPE for $T\{J(0)j_5(Y)\}$. The log q^2-terms reflect the fact that the lowest singularity in $\Phi^{PT}(q^2, Q^2, M^2)$ for the q-channel corresponds to threshold of the $q\bar{q}$-pair production which, for massless quarks, starts at zero. However, this is true only in perturbation theory. For hadrons, the first singularity in the q-channel is located at the $\pi\pi$ threshold, with the ρ-resonance being the most prominent feature of the physical spectral density for the correlators $\Pi_n^{(i)}(q^2)$. In other words, the $1/q^2$ and $1/q^4$ terms in the condensates and logarithmic terms in the perturbative contribution correspond to the operator product expansion for the correlators $\Pi_n^{(i)}(q^2)$, which is only valid in the large-q^2 region. To get $\Pi_n^{(i)}(q^2)$ for small q^2, one can use this large-q^2 information to construct a model for the physical spectral density $\sigma_n^{(i)}(t)$ and then calculate $\Pi_n^{(i)}(q^2)$ from the dispersion relation

$$\Pi_n^{(i)}(q^2) = \frac{1}{\pi} \int_0^\infty \frac{\sigma_n^{(i)}(t)dt}{t + q^2}. \qquad (17)$$

To this end, we use the usual "first resonance plus continuum" model

$$\sigma_n^{(i)}(t) = g_n^{(i)}\delta(t - m_\rho^2) + \theta(t > s_\rho)\sigma_n^{(i)PT}(t). \qquad (18)$$

Practically, this means that we modify the original spectral density in the region $t < s_\rho$ by subtracting all the terms of the OPE for $\sigma_n(t)$ in this region and replace them by the ρ-meson contribution $g_n^{(i)}\delta(t - m_\rho^2)$. In particular, this subtraction eliminates $1/q^2$ and $1/q^4$ singularities corresponding to the condensate $\delta(t)$- and $\delta'(t)$-contributions into $\sigma_n(t)$. For the perturbative

contribution, the subtraction procedure removes the $\log q^2$ terms. If $\sigma^{PT}(t) \sim t$ (this produces the $q^2 \log q^2$ contribution), the "correction term" is given by

$$-\int_0^{s_\rho} \frac{t\,dt}{t+q^2} + \frac{g^{(1)}}{q^2+m_\rho^2} = -s_\rho + q^2 \log\left(\frac{s_\rho+q^2}{q^2}\right) + \frac{g^{(1)}}{q^2+m_\rho^2}. \tag{19}$$

As a result, the $-q^2 \log q^2$ term cancels the first non-analytic term in the three-point function, and effectively one gets $\log s_\rho$ instead of $\log q^2$ in the small-q^2 region. In the large-q^2 region, where the original OPE must be valid, the correction terms should disappear. Requiring that they vanish there faster than the contribution which they are correcting (*i.e.*, faster than $1/q^2$) we arrive at the relation $g^{(1)} = s_\rho^2/2$. In a similar way, if $\sigma^{PT}(t) \sim t^2$ (this gives the $q^4 \log q^2$ term) one gets the relation $g^{(2)} = -s_\rho^3/3$.

Imposing a universal n-independent prescription is equivalent to the assumption that the y-dependence of the ρ-meson distribution amplitudes coincides with that of the perturbative correlators and, furthermore, that the ρ-meson contribution is dual to the quark one, with the standard duality interval $s_\rho \approx 1.5\,GeV^2$ obtained in ref.[11]. In principle, one can use more elaborate models for the distribution amplitudes $\varphi_\rho^{(i)}(y)$. As far as the requirement of smallness of the additional terms in the large-q^2 region is fulfilled, our results do not show strong sensitivity to particular forms of $\varphi_\rho^{(i)}(y)$. However, using the local duality approximation for $\varphi_\rho^{(i)}(y)$ we are able to present our results in a more compact and explicit form.

After the modifications described above, the contribution of the triangle diagram converts into

$$\Phi_0(M^2, q^2, Q^2) = \frac{1}{\pi M^2} \int_0^\infty ds\, e^{-s/M^2} \frac{Q^2}{(s+Q^2)^2} \Big\{ 1 + e^{yq^2/M^2}$$
$$\left[2\frac{y}{M^2} \left(q^2 \ln \frac{y(s_\rho+q^2)}{M^2} - s_\rho + \frac{s_\rho^2}{2(m_\rho^2+q^2)} \right) \right.$$
$$\left. + \frac{y^2}{M^4} \left(q^4 \ln \frac{y(s_\rho+q^2)}{M^2} - q^2 s_\rho + \frac{s_\rho^2}{2} - \frac{s_\rho^3}{3(m_\rho^2+q^2)} \right) \right] + \dots \Big\} \tag{20}$$

(here $y = s/(s+Q^2)$). As promised, in this expression, we can take the limit $q^2 \to 0$ without encountering any non-analytic terms. Note, that the modified versions of $q^2 \log q^2$ and $q^4 \log q^2$ terms do not vanish in the $q^2 \to 0$ limit. Finally, using the formula

$$\int_0^\infty e^{-s/M^2} g(s) \frac{ds}{M^6} = \int_0^\infty e^{-s/M^2} (g(0)\,\delta(s) + g'(s)) \frac{ds}{M^4}$$
$$= \int_0^\infty e^{-s/M^2} (g'(0)\,\delta(s) + g(0)\,\delta'(s) + g''(s)) \frac{ds}{M^2} \tag{21}$$

we can rewrite the $q^2 \to 0$ limit of eq.(20) in the canonical form (8) and determine the relevant spectral density $\rho_0(s, Q^2)$.

Most of the singular condensate contributions of the original sum rule (10) can be interpreted in terms of the bilocals corresponding to the simplest, one-propagator coefficient function. As a result, they are subtracted by the procedure described above. To factorize the gluon condensate

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contribution, we used a technique similar to that developed for the perturbative term. For the diagrams with the quark condensate, the factorization in most cases is trivial. However, there are also terms with the coefficient function formed by three propagators (see Fig.2b). In this case, the long-distance contribution is described by the photon distribution amplitude $\phi^T(y, q^2)$ related to the $\mathcal{O} \sim \bar{q}\gamma_5[\gamma_\alpha, \gamma_\beta]D \ldots Dq$ operators. The OPE for such a "non-diagonal" correlator (cf. [21]) starts with the term proportional to the quark condensate: $\phi^T(y, q^2) \sim \frac{1}{q^2}\langle\bar{q}q\rangle[\delta(y) + \delta(1 - y)]$, strongly peaked at the end-points. However, incorporating the nonlocal condensates to model the higher-dimension contributions and employing a novel technique [22] applied earlier to a similar non-diagonal correlator, we found that the distribution amplitude of the lowest-state (*i.e.*, ρ-meson) is rather narrow. Neglecting the higher-state contributions (whose distribution amplitudes have oscillatory form) we obtain in the $q^2 = 0$ limit

$$\Phi^T(M^2, Q^2) = \frac{128\pi^2\alpha_s\langle\bar{q}q\rangle^2}{27m_\rho^2 Q^4 M^6} \int_0^\infty e^{-s/M^2} s\,ds \int_{s/(s+Q^2)}^1 \frac{\varphi_\rho^T(y)}{y^2}dy, \tag{22}$$

where $\varphi_\rho^T(y)$ is the normalized distribution amplitude (its zeroth y-moment equals 1), which we model by $\varphi_\rho^T(y) = 6y(1 - y)$. Transforming $\Phi^T(M^2, Q^2)$ to the canonical form by using eq.(21) gives the spectral density $\rho^T(s, Q^2)$. Note that, for our model, $\rho^T(s, Q^2)$ contains the $(1/s)_+$ distribution.

The OPE for the three-propagator coefficient function also produces operators like $\bar{q}\ldots\gamma_\mu D^\mu q$. Naively, one would expect that such operators vanish due to equations of motion. However, when inserted in a correlator, they produce the so-called contact terms [23]. In our case, the contact terms give

$$\Phi^C(Q^2, M^2) = -\frac{256\pi^2\alpha_s\langle\bar{q}q\rangle^2}{27Q^6 M^6} \int_0^\infty e^{-s/M^2} \left[\ln\frac{s + Q^2}{s} - 2\frac{Q^2}{s + Q^2}\right] s\,ds. \tag{23}$$

Again, the spectral density $\rho^C(s, Q^2)$, obtained after applying eq.(21), contains the $(1/s)_+$ distribution.

Finally, there are also configurations with the coefficient functions given by two propagators (see Fig.2c) which correspond to three-body $\bar{q}Gq$-type distribution amplitudes. Their contribution was found to be small and, to simplify the presentation, we will not include them here.

6.   Sum rule for $F_{\gamma^*\gamma^*\pi^0}(q^2, Q^2)$ in the $q^2 = 0$ limit.

Since all the contributions, which were singular or non-analytic in the small-q^2 limit of the original sum rule, are now properly modified, we can take the limit $q^2 = 0$ and write down our QCD sum rule for the $\gamma\gamma^* \to \pi^0$ form factor:

$$\pi f_\pi F_{\gamma\gamma^*\pi^0}(Q^2) = \int_0^{s_0} \left\{1 - 2\frac{Q^2 - 2s}{(s + Q^2)^2}\left(s_\rho - \frac{s_\rho^2}{2m_\rho^2}\right)\right.$$

$$
\begin{aligned}
+ \quad & 2\frac{Q^2 - 6s + 3s^2/Q^2}{(s + Q^2)^4}\left(\frac{s_\rho^2}{2} - \frac{s_\rho^3}{3m_\rho^2}\right)\bigg\} e^{-s/M^2}\frac{Q^2 ds}{(s + Q^2)^2} \\
+ \quad & \frac{\pi^2}{9}\langle\frac{\alpha_s}{\pi}GG\rangle\left\{\frac{1}{2Q^2 M^2} + \frac{1}{Q^4} - 2\int_0^{s_0} e^{-s/M^2}\frac{ds}{(s + Q^2)^3}\right\} \\
+ \quad & \frac{64}{27}\pi^3\alpha_s\langle\bar{q}q\rangle^2 \lim_{\lambda^2\to 0}\left\{\frac{1}{2Q^2 M^4} + \frac{12}{Q^4 m_\rho^2}\left[\log\frac{Q^2}{\lambda^2} - 2\right.\right. \\
+ \quad & \left.\int_0^{s_0} e^{-s/M^2}\left(\frac{s^2 + 3sQ^2 + 4Q^4}{(s + Q^2)^3} - \frac{1}{s + \lambda^2}\right)ds\right] \\
- \quad & \frac{4}{Q^6}\left[\log\frac{Q^2}{\lambda^2} - 3 + \int_0^{s_0} e^{-s/M^2}\left(\frac{s^2 + 3sQ^2 + 6Q^4}{(s + Q^2)^3} - \frac{1}{s + \lambda^2}\right)ds\right]\bigg\}. \quad (24)
\end{aligned}
$$

The sum rule must be taken in the limit $\lambda^2 \to 0$ of the parameter λ^2 specifying the regularization which we used to calculate the integrals with the $(1/s)_+$ distribution. Furthermore, this sum rule implies that the continuum is modeled by an effective spectral density $\rho^{eff}(s, Q^2)$ rather than by $\rho^{PT}(s, Q^2)$, with $\rho^{eff}(s, Q^2)$ including all the spectral densities which are nonzero for $s > 0$, i.e., $\rho_0(s, Q^2)$, $\rho^T(s, Q^2)$, $\rho^C(s, Q^2)$ and also an analogous contribution from the gluon condensate term.

Using the standard values for the condensates and ρ-meson duality interval $s_\rho = 1.5\,GeV^2$, [11], we studied the stability of the sum rule with respect to variations of the SVZ-Borel parameter M^2 in the region $M^2 > 0.6\,GeV^2$. Good stability was observed not only for the "canonical" value $s_0^\pi = 4\pi^2 f_\pi^2 \approx 0.7\,GeV^2$ but also for smaller values of s_0, even as small as $0.4\,GeV^2$. Since our results are sensitive to the s_0-value, we incorporated a more detailed model for the spectral density, treating the A_1-meson as a separate resonance at $s = 1.7\,GeV^2$, with the continuum starting at some larger value s_A. The results obtained in this way have good M^2-stability and, for $M^2 < 1.2\,GeV^2$, show no significant dependence on s_A. Numerically, they practically coincide with the results obtained from the sum rule (24) for $s_0 = 0.7\,GeV^2$.

In Fig.3, we present a curve for $Q^2 F_{\gamma\gamma^*\pi^0}(Q^2)/4\pi f_\pi$ calculated from eq.(24) for $s_0 = 0.7\,GeV^2$ and $M^2 = 0.8\,GeV^2$. It is rather close to the curve corresponding to the Brodsky-Lepage interpolation formula $\pi f_\pi F_{\gamma\gamma^*\pi^0}(Q^2) = 1/(1 + Q^2/4\pi^2 f_\pi^2)$ and to that based on the ρ-pole approximation $\pi f_\pi F(Q^2) = 1/(1 + Q^2/m_\rho^2)$. It should be noted, however, that the closeness of our results to the ρ-pole behaviour in the Q^2-channel has nothing to do with the explicit use of the ρ-contributions in our models for the correlators in the q^2-channel: the Q^2-dependence of the ρ-pole type emerges due to the fact that the pion duality interval $s_0 \approx 0.7\,GeV^2$ is numerically close to $m_\rho^2 \approx 0.6\,GeV^2$.

For $Q^2 < 3\,GeV^2$, our curve goes slightly above those based on the ρ-pole dominance and BL-interpolation (which are close to the data [9]). This overshooting is a consequence of our assumption that Q^2 can be treated as a large variable: in some terms, $1/Q^2$ serves as an expansion parameter. Such an approximation for these terms is invalid for small Q^2 and appreciably overestimates them for $Q^2 \sim 1\,GeV^2$ producing enlarged values for $F_{\gamma\gamma^*\pi^0}(Q^2)$.

In the region $Q^2 > 3\,GeV^2$, our curve for $Q^2 F_{\gamma\gamma^*\pi^0}(Q^2)$ is practically constant, supporting the pQCD expectation (1). The absolute magnitude of our prediction gives $I \approx 2.4$ for the I-integral.
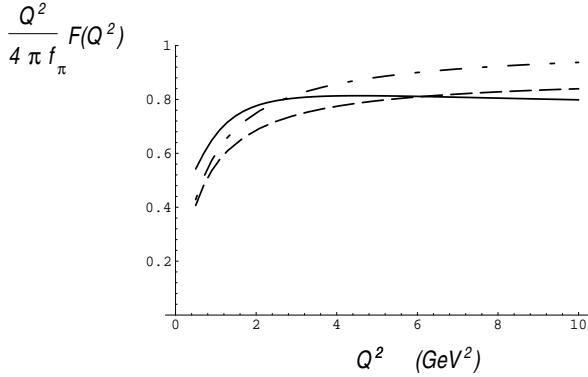
Figure 3: Combination $Q^2 F_{\gamma\gamma^*\pi^0}(Q^2)/4\pi f_\pi$ as calculated from the QCD sum rule (solid line), ρ-pole model (dashed line) and Brodsky-Lepage interpolation (dash-dotted line).

Of course, this value has some uncertainty: it will drift if we change our models for the photon distribution amplitudes (bilocals). The strongest sensitivity is to the choice of $\varphi_\rho^T(y)$ in the tensor contribution (22). However, even rather drastic changes in the form of $\varphi_\rho^T(y)$ do not increase our result for I by more than 20%. The basic reason for this stability is that the potentially large $1/q^2$ factor from the relevant contribution in the original sum rule (10) is substituted in (22) by a rather small (and non-adjustible) factor $1/m_\rho^2$.

Comparing the value $I = 2.4$ with $I^{as} = 3$ and $I^{CZ} = 5$, we conclude that our result favours a pion distribution amplitude which is narrower than the asymptotic form. Parametrizing the width of $\varphi_\pi(x)$ by a simple model $\varphi_\pi(x) \sim [x(1-x)]^n$, we get that $I = 2.4$ corresponds to $n = 2.5$. The second moment $\langle \xi^2 \rangle$ (ξ is the relative fraction $\xi = x - \bar{x}$) for such a function is 0.125. This low value (recall that $\langle \xi^2 \rangle^{as} = 0.2$ while $\langle \xi^2 \rangle^{CZ} = 0.43$) agrees, however, with the lattice calculation [24] and also with the recent result [22] obtained from the analysis of a non-diagonal correlator.

7. Conclusions.

Thus, the QCD sum rules support the expectation that the Q^2-dependence of the transition form factor $F_{\gamma\gamma^*\pi^0}(Q^2)$ is rather close to a simple interpolation between the $Q^2 = 0$ value (fixed by the ABJ anomaly) and the large-Q^2 pQCD behaviour $F(Q^2) \sim Q^{-2}$. Moreover, the QCD sum rule approach enables us to calculate the absolute normalization of the Q^{-2} term. The value produced by the QCD sum rule is close to that corresponding to the asymptotic form $\varphi_\pi^{as}(x) = 6 f_\pi x(1-x)$ of the pion distribution amplitude. Our curve for $F_{\gamma\gamma^*\pi^0}(Q^2)$ is also in satisfactory agreement with the CELLO data [9] and in good agreement with preliminary high-Q^2 results from CLEO [10]. Hence, there is a very solid evidence, both theoretical and experimental, that $\varphi_\pi(x)$ is a rather narrow function.

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